

B.Sc. EXAMINATION by Course Unit

MAS101 Calculus I

Thursday, 1 May 2003, 10.00–12.00

The duration of this examination is 2 hours.

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

SECTION A: Each question carries 8 marks. You should attempt ALL 5 questions.

SECTION B: Each question carries 20 marks. You may attempt all questions but, except for the award of a bare pass, only marks for the best 3 questions will be counted.

Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

SECTION A

Each question carries 8 marks. You should attempt ALL 5 questions.

1. Let $y = e^{3x} \sin(5x)$. Find $\frac{d^3y}{dx^3}$.
2. Find all solutions in the complex numbers of the following equations:
 - (i) $z^2 - (5 + 9i)z - 14 + 23i = 0$
 - (ii) $z^2 + (5 + 9i)z - 14 + 23i = 0$
 - (iii) $z^2 - (5 - 9i)z - 14 - 23i = 0$
 - (iv) $z^2 + (5 - 9i)z - 14 - 23i = 0$

Briefly explain why it is easy to deduce the solutions in cases (ii), (iii) and (iv) from case (i), without the need to do further calculation.

3. Draw a sketch of the complex plane (Argand diagram) including the following subsets:
 - $\{z \in \mathbb{C}; |z + 4 - i| > 3\}$
 - $\{z \in \mathbb{C}; |z + 2 - 2i| \geq |z + 1|\}$
4. What is the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 34y = 0?$$

5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \int_{\cosh x}^{\cosh x} \frac{1}{\sqrt{1+t^2}} dt.$$

Find an expression for $f'(x)$.

SECTION B

Each question carries 20 marks. You may attempt all questions but, except for the award of a bare pass, only marks for the best 3 questions will be counted.

1. Find the general solutions of the differential equations

$$\frac{dy}{dx} - 7y = (1 + e^{7x}) \cos(2x),$$

and

$$\frac{dy}{dx} - 7y = (1 + e^{7x}) \sin(2x).$$

2. Compute the following Riemann integrals.

(i) $\int_{-1}^1 (\sinh x)^7 dx$

(ii) $\int_0^\pi (\cos x)^3 dx$

(iii) $\int_2^3 (\ln x)^3 dx$

3. State the Mean Value Theorem.

Prove that if f is a differentiable function from $\mathbb{R}$ to $\mathbb{R}$ such that $|f(x)| < 1$ for all x , then there exists a real number t between 0 and 1000 for which $|f'(t)| < \frac{1}{500}$.

Give an example of such an f with the property that $f'(x) < 0$ for all x .

4. (a) State Taylor's Theorem and write down the first four non-zero terms of the Taylor series for

(i) $\cos(x^4)$ about $x = 0$

(ii) $\ln(1 + x^2)$ about $x = 0$

What are the radii of convergence of these two Taylor series?

- (b) By using l'Hôpital's rule or otherwise, determine the following limits:

(i) $\lim_{x \rightarrow 0} \frac{e^{-4x} - e^{-5x}}{x}$

(ii) $\lim_{x \rightarrow \pi} \frac{\sin(5x)}{\sin(3x)}$

(iii) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

(iv) $\lim_{x \rightarrow 0} x^8 e^{-1/x^2}$

5. State De Moivre's Theorem and use it to express

(i) $\cos(4x)$ as a polynomial function of $c := \cos x$; and

(ii) $\tan(4x)$ as a rational function of $t := \tan x$.